FAR BEYOND

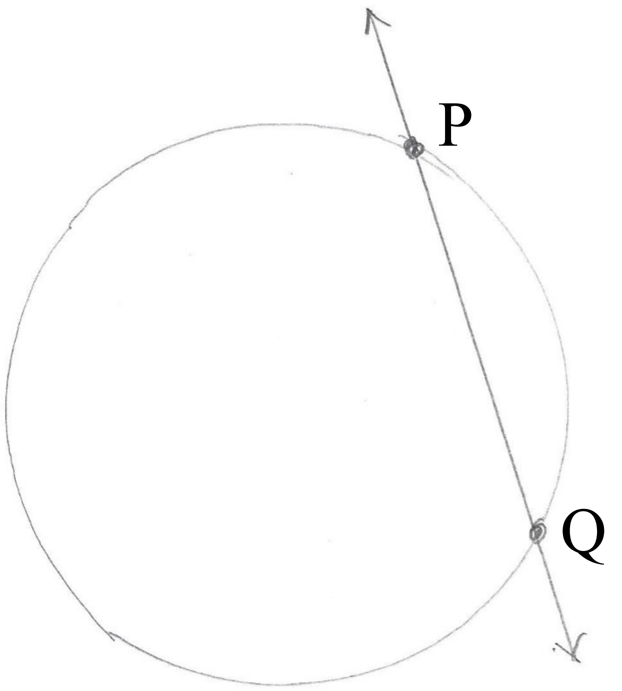
MAT122

Rate of Change

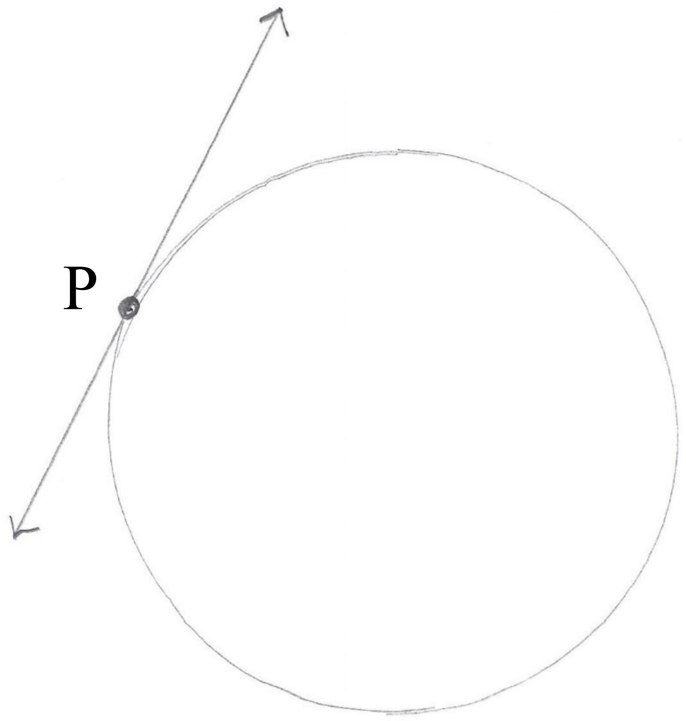
Stony Brook University

Secant vs Tangent on a Curve

Recall:



secant line

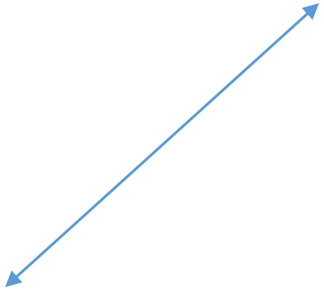


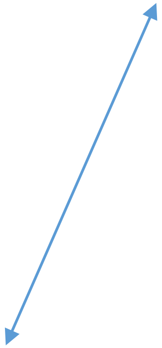
tangent line

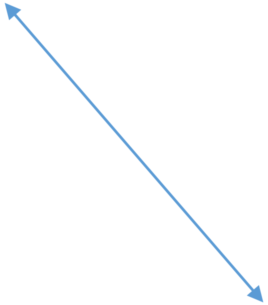
Rate of Change of a Line - Refresher

For a linear function, slope measures the **steepness**

slope is also its **rate of change**







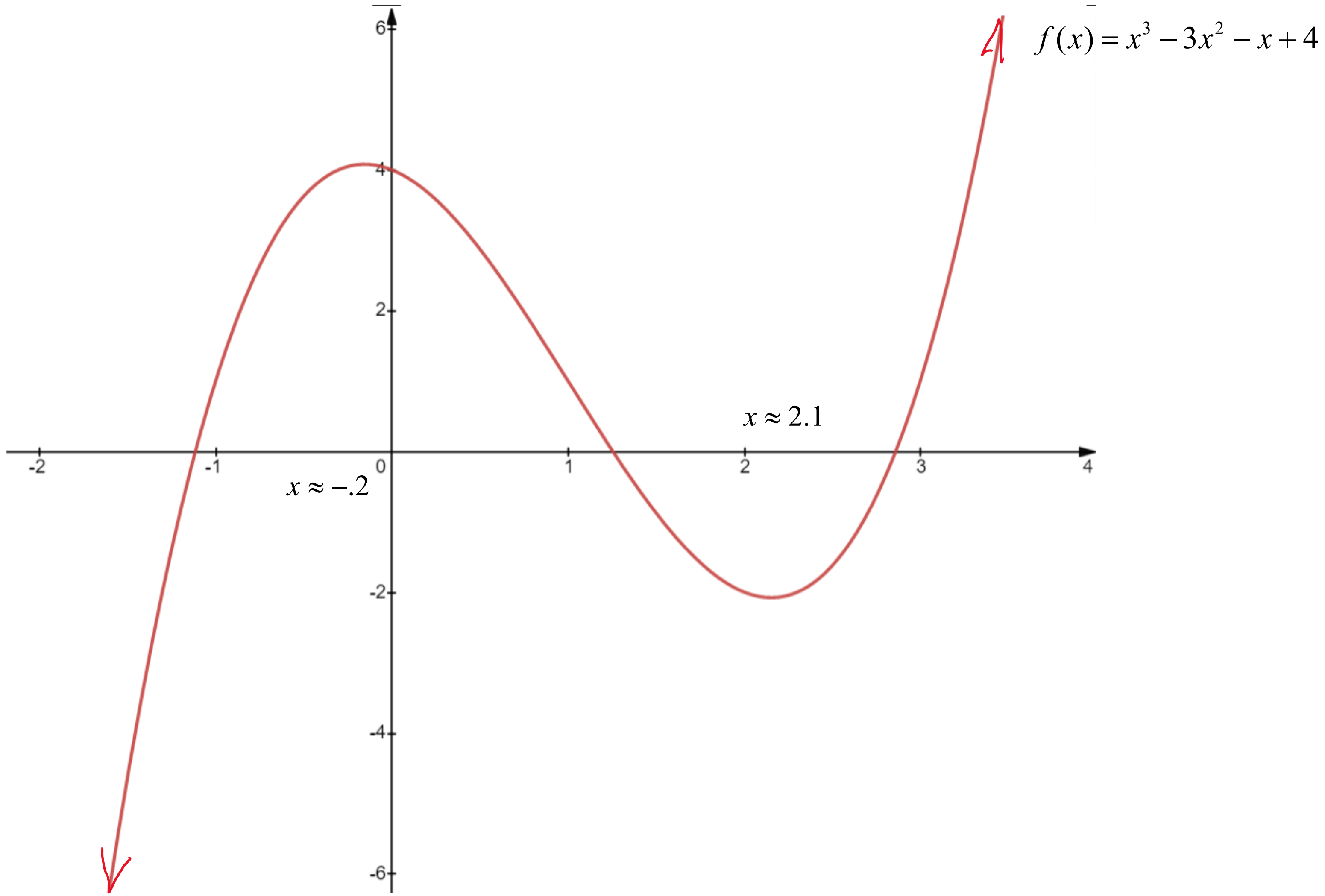
slope is 0

(*x*-values and *y*-values are both increasing)

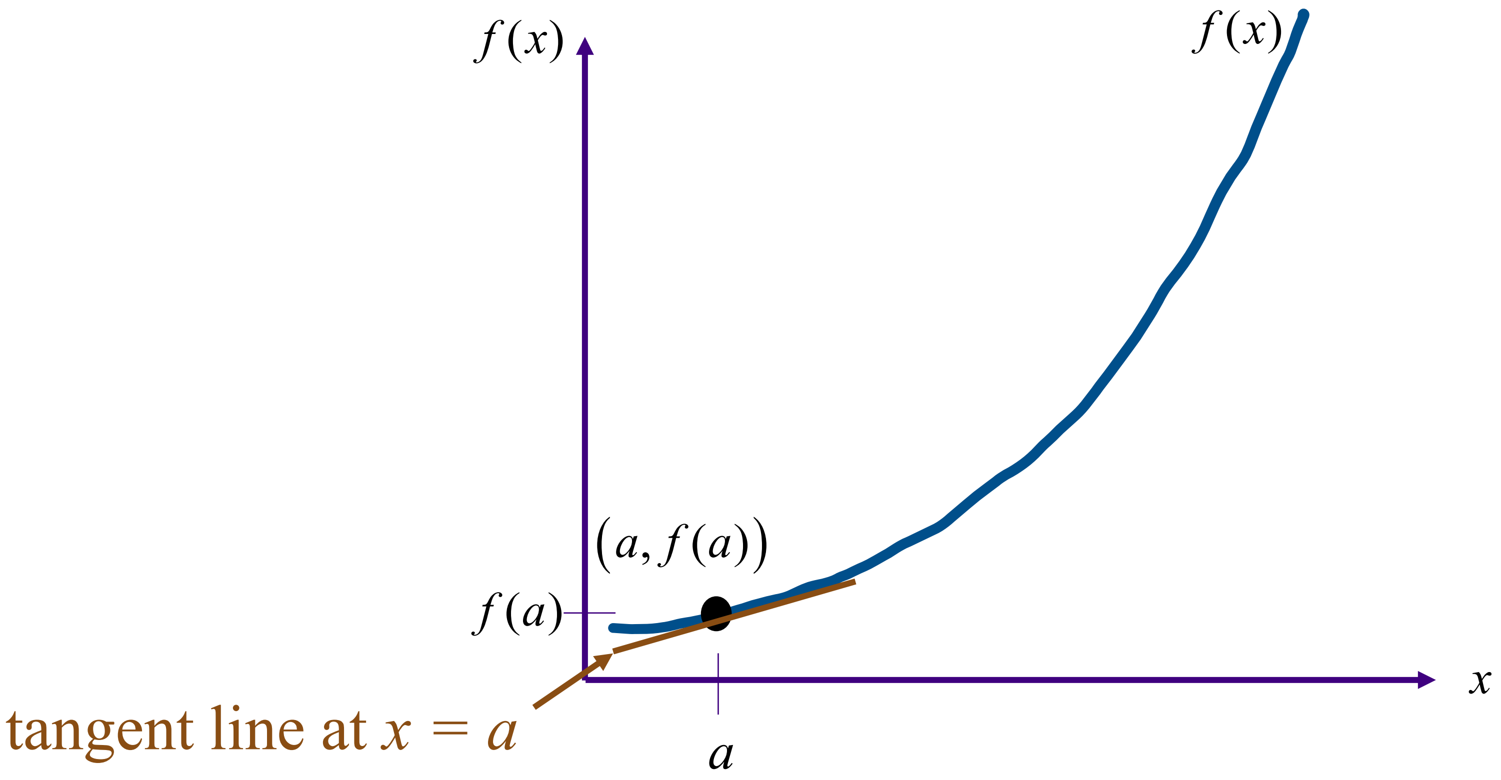
(as *x*-values are increasing,
y-values are decreasing)

Rate of Change on a Curve

Contrary to a linear function, the rate of change can vary at different places on the curve.



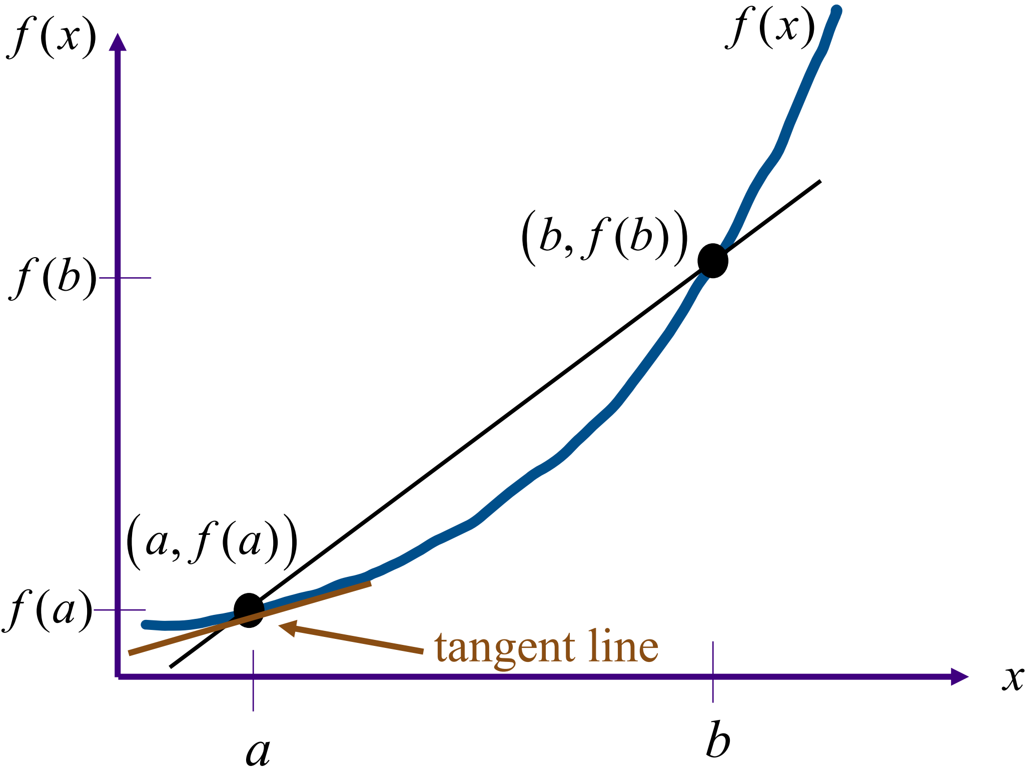
Problem Finding Slope of Tangent Line



Problem: no way to find a slope if only *one* ordered pair is known

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Average vs. Instantaneous Rate of Change

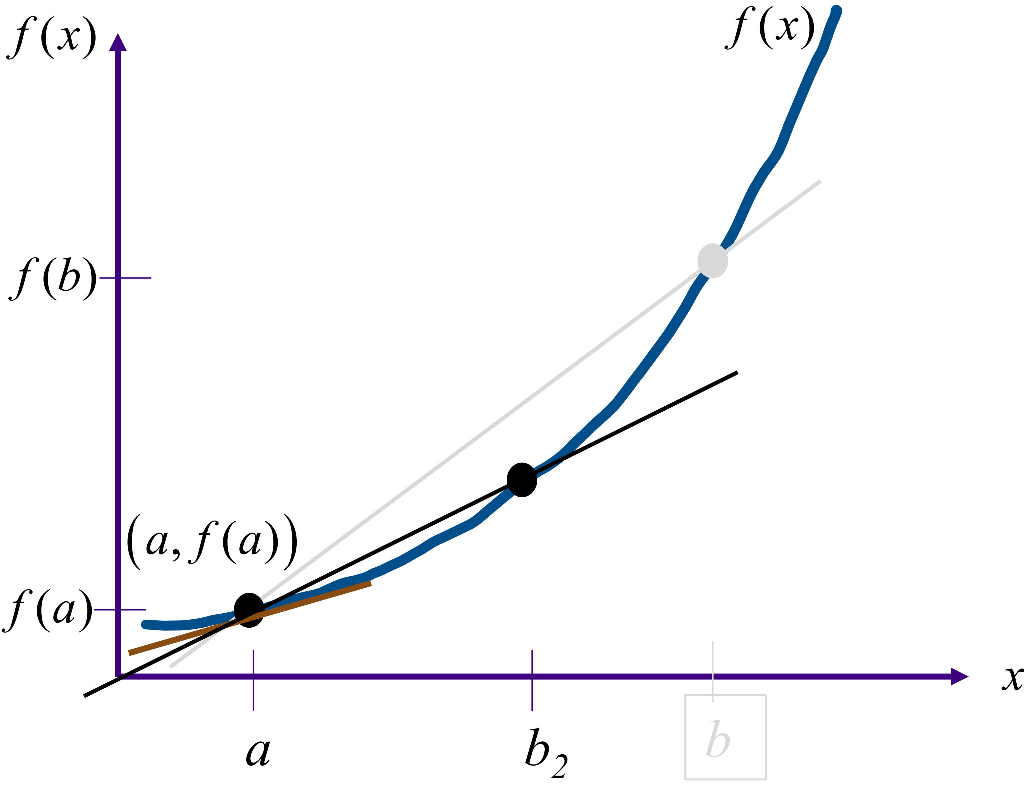


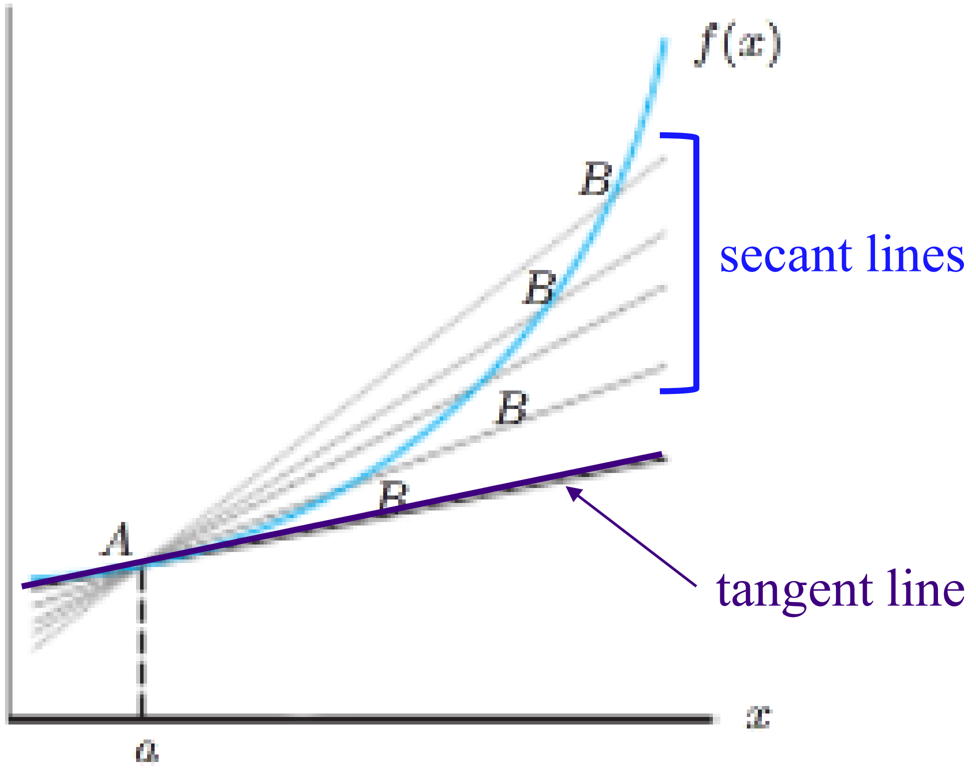
slope of the secant line <u>estimates</u>
the rate of change at $(a, f(a))$

called **<u>average</u>** rate of change at that point

Fact: estimate will be better the closer that b is to a

How close is "nearby"?

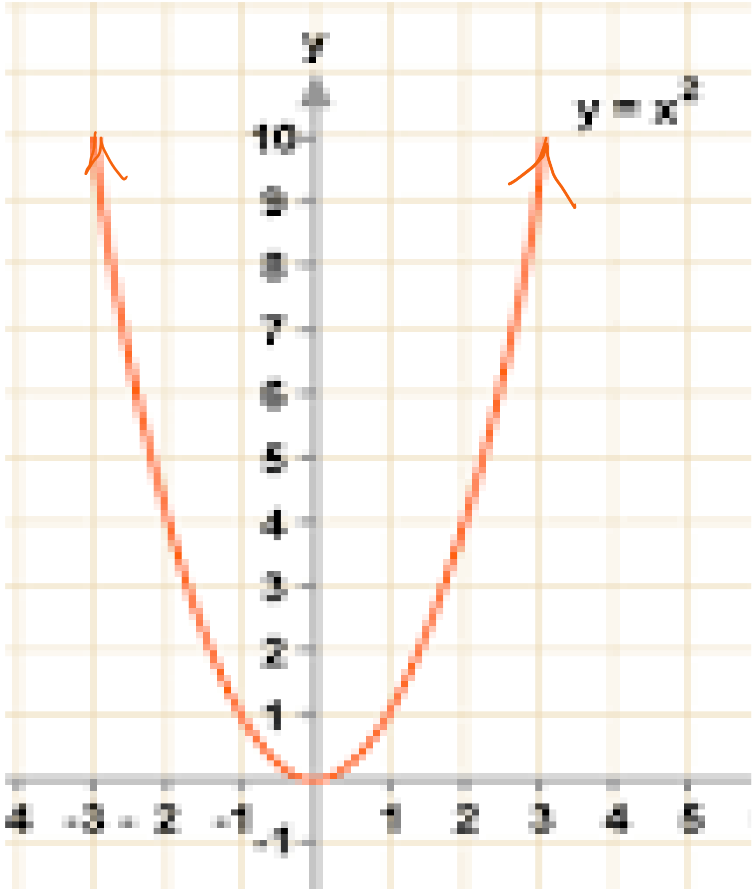




slope of tangent line is exact rate of change at $x = a$

Identify Rates of Change on a Graph

The **derivative** of f at $x = a$, written as $f'(a)$, is the instantaneous rate of change of f at $x = a$.



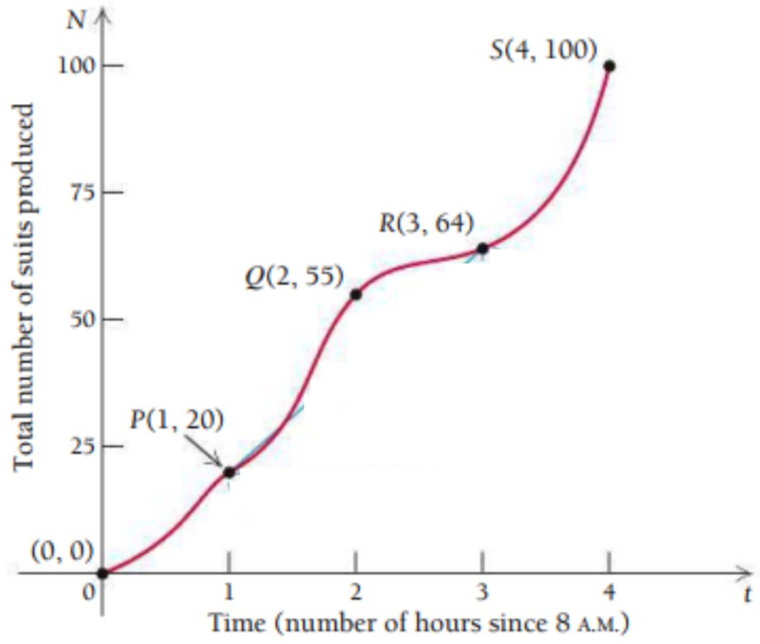
ex. Determine if the following are positive, negative or zero:

- $f'(1)$
- $f'(-1)$
- $f'(2)$
- $f'(0)$

Rates of Change - Application

ex. How many suits were produced between 9am and 10am?

ex. What is the slope of the secant line between $t = 1$ and $t = 2$?



ex. What was the average number of suits produced per hour from 9am to 11am?

More Rates of Change on a Graph

derivative $= f'(a) =$ slope of tangent line at $x = a$

ex. Given a graph, illustrate the following graphically and determine if positive or negative.

- $f'(1)$
- $\dfrac{f(2)-f(1)}{2-1}$
- $f(4)-f(2)$